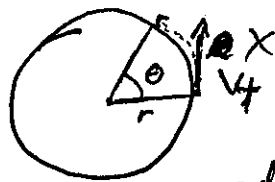


Ch. 8: Rotational Motion

Angular Quantities



$$X = r\theta$$

$$\text{rad} \rightarrow \text{deg}: \frac{360}{2\pi} \text{ or } \frac{180}{\pi}$$

$$\text{deg} \rightarrow \text{rad}: \frac{2\pi}{360} \text{ or } \frac{\pi}{180}$$

For a full circle, $\theta = 2\pi = 360^\circ$

$$\text{circumference} = X = r(2\pi) = 2\pi r$$

For angular motion, we measure distance typically with angles.

Linear:

Angular:

$$X \rightarrow \theta$$

Likewise, our linear velocity, v , becomes an angular velocity:

$$V_t = \frac{\Delta X}{\Delta t} = \frac{X_f - X_i}{t_f - t_i} \rightarrow \omega = \frac{\Delta \theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i} \quad \text{Units: } \left(\frac{\text{rad}}{\text{s}}\right)$$

Notice:



is tangential, moving around the circle rather than in/out (radial)

$$X = r\theta \Rightarrow \Delta X = r \Delta \theta$$

$$\Rightarrow \frac{\Delta X}{\Delta t} = r \frac{\Delta \theta}{\Delta t}$$

$$\Rightarrow \boxed{V_t = r\omega}$$

Note: Frequency:



$$\text{if } v = \frac{\text{distance}}{\text{time}} \Rightarrow \frac{1}{\text{time}} = \frac{v}{\text{distance}}$$

$$\frac{1}{T} = \frac{V_t}{2\pi r} = \frac{r\omega}{2\pi r} = \frac{\omega}{2\pi} \Rightarrow \boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$

Ch. 2:

For acceleration, we see the same result:

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \rightarrow \alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} \quad \left(\frac{\text{rad}}{\text{s}^2}\right)$$

and: $v_t = r\omega \Rightarrow \Delta v_t = r \Delta \omega$

$$\Rightarrow \frac{\Delta v_t}{\Delta t} = r \frac{\Delta \omega}{\Delta t}$$

$$\Rightarrow \boxed{a_t = r \alpha}$$



Recall: If we're moving in a circle, we also have some centripetal acceleration:

$$a_r = \frac{v_c^2}{r}$$

Which we can now note that $v \rightarrow v_t$
and thus:

$$a_r = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

$$\boxed{a_r = \omega^2 r}$$



Note: Understand the difference!!!

~~Questions~~ Questions: ~~1, 2~~ Together: 1, 2,

Problems: 1+3, 6 (Note: 1 rev = 2π rad), 9 (What is period/radians for each?)

$$3:] r = 380\,000\,000$$

$$\theta = 1.4 \times 10^{-5} \text{ rad}$$

$$x = r\theta = 5.3 \times 10^3 \text{ m}$$

$$6.] r = .34 \text{ m}$$

$$d = 8000 \text{ m}$$

$$d = r\theta \Rightarrow \theta = \frac{d}{r} = 23529 \text{ div by } 2\pi = \boxed{3.7 \times 10^3 \text{ rev}}$$

$$9.] \omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{1 \text{ yr}} = \frac{2\pi}{(3.16 \times 10^7 \text{ s})} = 1.99 \times 10^{-7} \frac{\text{rad}}{\text{s}}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{1 \text{ d}} = \frac{2\pi}{86,400 \text{ s}} = 7.27 \times 10^{-5} \frac{\text{rad}}{\text{s}}$$

Ch. 8: ANGULAR KINEMATICS

Recall: $\begin{array}{c} \text{Linear} \\ x, y \end{array} \rightarrow \begin{array}{c} \text{Angular} \\ \theta \end{array}$

$$v \rightarrow \omega$$

~~Recall:~~ $a \rightarrow \alpha$

Also recall Linear Kinematics Equations (constant acceleration)

Linear

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} at^2$$

~~$v^2 = v_0^2 + 2ax$~~

$$v^2 = v_0^2 + 2ax$$

$$\bar{v} = \frac{v + v_0}{2}$$

Angular

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\bar{\omega} = \frac{\omega + \omega_0}{2}$$

Ex: ^{Prob:} 15.) $\omega = 15000 \text{ rpm} \cdot \frac{2\pi}{60s} ; t = 220s$

$$\omega = \omega_0 + \alpha t \Rightarrow \omega = \alpha t \Rightarrow \alpha = \frac{\omega}{t}$$

$$\Rightarrow \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow \theta = \frac{1}{2} \left(\frac{\omega}{t} \right) t^2 = \frac{1}{2} \omega t = 172780 \text{ rad}$$

$$\frac{\theta}{2\pi} = \boxed{2.7 \times 10^4 \text{ rev}}$$

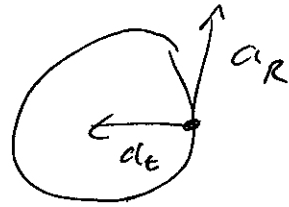
$$t' = 2.0 \text{ s}$$

7.]

$$\alpha = \frac{\omega_f' - \omega_0}{t'} \Rightarrow \omega_f' = \omega_0 + \alpha t' = 21.5 \text{ rad/s}$$

$$a_c = \omega^2 r = 1.6 \times 10^2 \text{ m/s}^2$$

$$a_t = dr = 1.4 \text{ m/s}^2$$



$$1.) \quad 45^\circ \cdot \frac{\pi \text{ rad}}{180^\circ} = \boxed{\frac{\pi}{4} \text{ rad}}$$

2.) - KE is conserved for ^{COMPLETELY} ELASTIC ONLY.

- Complete inelastic means sticking
- momentum always conserved
- collision!

$$3.) \quad m_1 = .230 \text{ kg} \quad \rightarrow \quad \text{O} \rightarrow \text{O} \quad | \quad \text{O} \rightarrow$$

$$m_2 = .350 \text{ kg}$$

$$v = .28 \text{ m/s}$$

$$(m_1 + m_2) v = 0.16 \text{ kg m/s}$$

4.) Law of conservation of momentum

$$5.) \quad p_x = m_1 v_x \Rightarrow v_x = \frac{p_x}{m_1} = 0.70 \text{ m/s}$$

$$6.) \quad r = .35 \text{ m} \quad t_f = 4.0 \text{ s}$$

$$\omega_f = 280 \text{ rpm} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 29.5 \frac{\text{rad}}{\text{s}}$$

$$\omega_o = 136 \text{ rpm} = 13.6 \frac{\text{rad}}{\text{s}}$$

$$\alpha = \frac{\Delta \omega}{\Delta t} = 3.9 \text{ rad/s}^2$$

Rotational Kinetic Energy

Recall: Kinetic Energy is "energy of motion"

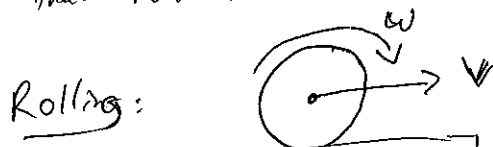
Non rotational KE is "translational"

$$KE_{\text{trans}} = \frac{1}{2} M V^2$$

If object is rotating, it is still in motion - "rotational motion"

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

Objects that roll ~~over~~ can have both!



Note: $v = r\omega$

~~Rolling~~

$$KE = \frac{1}{2} M V^2 + \frac{1}{2} I \omega^2$$

Angular Momentum

Recall Linear Momentum:

$$\vec{p} = m\vec{v}$$

A product of mass and velocity.

It follows that "angular momentum" would be a product of "angular mass" and "angular velocity"

Inertia

$$\vec{L} = I \vec{\omega}$$

Units: $\text{kg} \cdot \frac{\text{m}^2}{\text{s}}$

If $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ then it follows that $\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$

Recall: $\vec{p}_i = \vec{p}_f$

Likecase
~~then~~

$$\vec{L}_i = \vec{L}_f$$

Cons. of Momentum.

Cons. of Ang. Mom.

Recall,

$$\vec{L}_f = \vec{L}_i$$

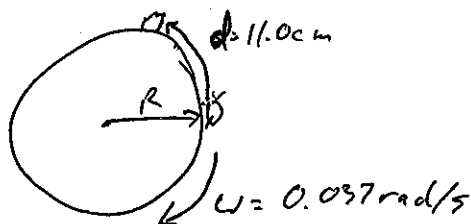
$$I_f \vec{\omega}_f = I_i \vec{\omega}_i$$

Star Question: 1.) $f = \frac{1}{T}$

2.) $\omega = 2\pi f$

3.) $v = \omega R$

Lazy Susan:



1.) $I_{\text{disk/cylinder}} = \frac{1}{2} MR^2 = 0.0027 \text{ kg} \cdot \text{m}^2$

How fast does cockroach walk relative to lazy susan?

$$\begin{aligned} d &= 11 \text{ cm} \\ t &= 3.2 \text{ s} \end{aligned} \quad V_{ls} = \frac{d}{t} = \boxed{0.034 \frac{\text{m}}{\text{s}}}$$

Speed of point on lazy susan:

$$V = \omega R = 0.0093 \frac{\text{m}}{\text{s}}$$

Speed of roach to earth?

$$V_e = V_{ls} - V = 0.025 \frac{\text{m}}{\text{s}}$$

~~Q66~~

Lazy Susan Ang. Mom.:

$$L_{ls} = I_{ls} \omega = 1 \times 10^{-4} \frac{\text{kg m}^2}{\text{s}}$$

Cock reach:

$$L = L_{ls} + L_c = 0$$

$$L_{ls} = -L_c =$$

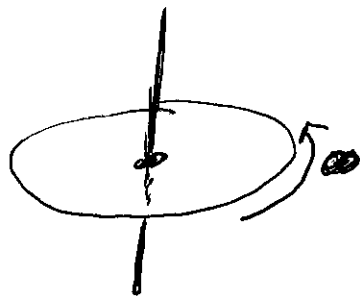
Mass:

$$L_{cr} = I_{cr} \omega_{cr} = (mR^2) \left(\frac{v}{R} \right) = 1 \times 10^{-4}$$

$$m: \frac{L_{cr}}{Rv} = 0.016 \text{ kg}$$

Angular Velocity + Angular Momentum

ω , α , L all point along the axis of rotation



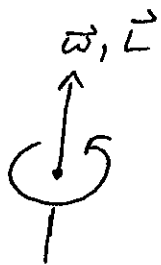
Only one direction is unique, ~~and~~ to the rotation, and that is along the axis.

We define a "right hand rule."

Fingers ^{follow} the path of rotation.

Curl fingers around.

Thumb points in direction of vector.



Note: $\vec{L} = I \vec{\omega}$

so $\vec{L} \parallel \vec{\omega}$



Cockroach went one way.

Lazy Susan went the other

the cancelled!

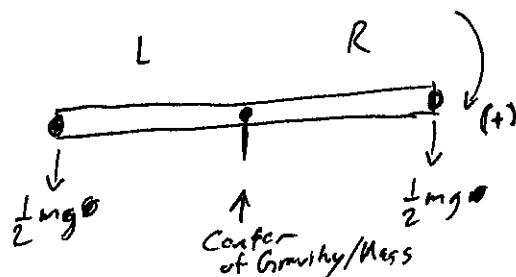
Equilibrium

Center of Mass / Center of Gravity

This is the point in an object where the object would balance (i.e., balance on a pin point).

This "balance" is ~~because~~ a result of the torques due to gravity cancelling out.

Balance point of Ruler demo.



$$\sum \tau = \frac{1}{2}mgr - \frac{1}{2}mgr = 0$$

Balanced. (thus, the halfway mark must be center of gravity.)

From here on out, when dealing with torques, we can ~~not~~ consider the entire weight of the object as acting through the CM/CG.

Erase Board!

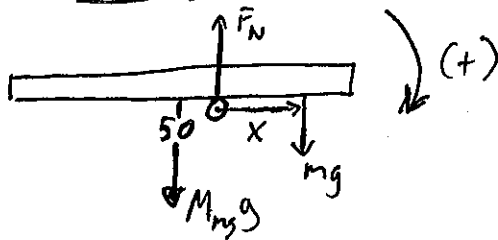
Example: Meter stick balance.

Center of Mass is at .50 m mark!

$$M_{ms} = 0.160 \text{ kg}$$

$$m = 0.050 \text{ kg}$$

Draw FBD!



Which dir. would each rotate?

$M_{ms}g$: CCW (-)

mg : CW (+)

F_N : N/A. Why?

Moment arms: $M_{ms}g$: $r_{ms} = (60\text{cm} - 50\text{cm}) = 0.10\text{m}$

mg : $r_m = x$

F_N : $r_N = 0$

Torques: $\tau_{ms} = -M_{ms}g r_{ms}$

$$\tau_m = mgx$$

$$\tau_N = 0$$

Balance? $\sum \tau = 0 \Rightarrow \sum \tau = -M_{ms}g r_{ms} + mgx = 0$

$$\Rightarrow x = \frac{M_{ms} r_{ms}}{m} = 32\text{cm.}$$

So: $60\text{cm} + 32\text{cm} = 92\text{cm}!$

Ch. 10 Fluids

Phases of Matter: 3 - Basic

Solid - fixed shape/size

Liquid - not fixed shape, but relatively fixed size.

Gas - no fixed shape, no fixed size.

liquids + gases can flow. we call them "fluids."

Any others?

Density: What is density?

Imagine a block of iron + block of wood.

same size; which would you want dropped on your head? Why?

- Iron is more dense!
- Iron has more mass per its size/volume!

$$\text{density: } \rho = \frac{m}{V} \quad \left[\frac{\text{kg}}{\text{m}^3} \right]$$

Sometimes we rewrite this to find mass:

$$m = \rho V$$

Note: p. 256 has a big table of densities.

Review
Volume Equations!!!
(check cover!)

Specific Gravity:

$$\text{ratio: } \frac{\text{density of substance}}{\text{density of water liquid}} = \frac{\text{density}}{1000 \text{ kg/m}^3}$$

Unitless.

Note: $\rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3}$; An object will float in a liquid that is more dense than it.

ice is less dense than water! (weird.)

Pressure

Pressure is the force acting on an area, or force per unit area!

$$P = \frac{F}{A} \quad \text{units: } \cancel{\text{Pa}} \text{ pascal (Pa) or } \text{N/m}^2$$

Imagine trying to push a hole into a wall using only your thumb. Now consider trying it using a thumb tack as well.

What's the difference?

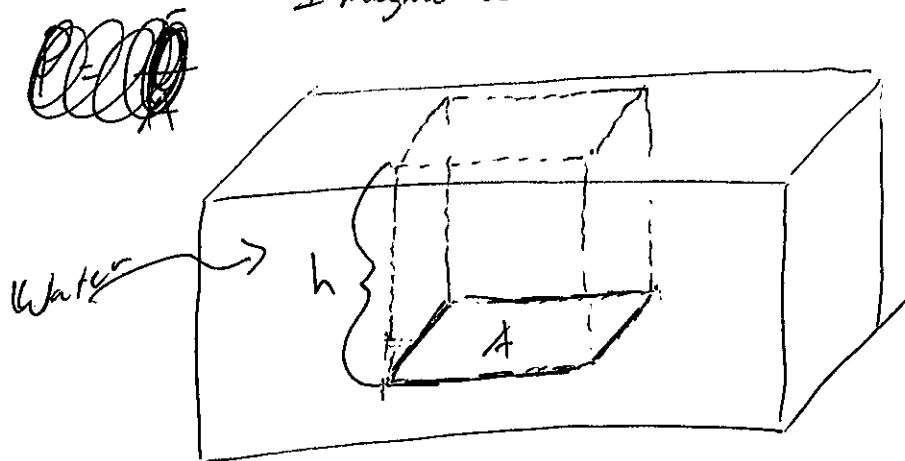
Similarly, how does, say, a soccer shin guard work?

~~Pressure~~

What happens when you swim deeper into water?

Why?

Imagine a board in the water



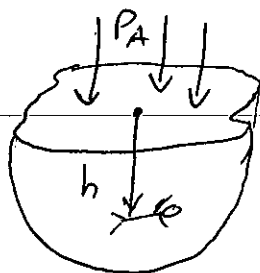
$$P = \frac{F}{A} = \frac{Mg}{A} = \frac{\rho Vg}{A} = \frac{\rho Ahg}{A}$$

$$\boxed{P = \rho gh}$$

Pressure at depth h .

Pascal's Principle ~~Absolute Pressure~~ Pressure change due to height.

If you're swimming under water the pressure you feel isn't only water pressure, but atmospheric + water pressure!



Atmosphere pushes on the water, which in turn pushes on you (along with the fluid pressure).

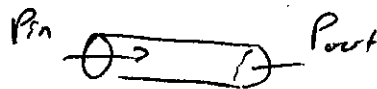
$$P = P_0 + \rho gh$$

Water tank Note: $\uparrow$ pressure on top, usually atmosphere. (P_A)
 Part b uses absolute pressure!!

Pascal's principle: "If an external pressure is applied to a confined fluid, then every point in the fluid will feel ~~an~~ ^{the} increased pressure."

$$P_{out} = P_{in}$$

or $\frac{F_{out}}{A_{out}} = \frac{F_{in}}{A_{in}}$



Hydraulic lift question:

$F_{in} \approx 800 \text{ N}$ $F_{out} = ?$
 $A_{in} = .25 \text{ m}^2$ $A_{out} = 1 \text{ m}^2$

$$\Rightarrow F_{out} = \frac{F_{in}}{A_{in}} \cdot A_{out} = 12800 \text{ N}$$

$$m \approx 1300 \text{ kg}$$

Atmospheric Pressure

- We previously discussed how as you go deeper in a fluid, the fluid's weight increases the pressure on you.
- We also previously discussed how gases, and not just liquids, are fluids.
- Think of Earth like a giant aquarium full of air instead of water, and we're at the bottom. There must be some pressure...

$$P_A = 1.013 \times 10^5 \text{ N/m}^2 = 1 \text{ atm}$$

How much is this?

Let's say we have a scale that is $.5 \times .5 \text{ m}^2$

$$P_A = \frac{F}{A} \Rightarrow F = P_A A$$

$$m = \frac{P_A (.25)}{g} = 2584 \text{ kg!!!}$$

How do we survive?

- The cells in our body match this pressure, like ~~air~~ air in an inflated balloon.

Straw slide Question

Note: Gauge Pressure vs. Absolute Pressure.

Gauge pressure is the pressure without atmosphere taken into account.
i.e. a tire gauge.

To find total, or Absolute pressure, we use:

$$P = P_A + P_G$$

~~If the tube changes in size~~

The flow rate should be the same on both sides of pipe.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Now we note that fluids are incompressible. We can squeeze as much as we want, but ρ won't change.

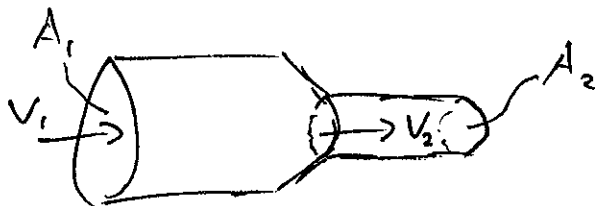
So: $\rho_1 = \rho_2 = \rho$

$$\cancel{\rho} A_1 v_1 = \cancel{\rho} A_2 v_2$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

Equation of Continuity

~~Continuity Principle~~

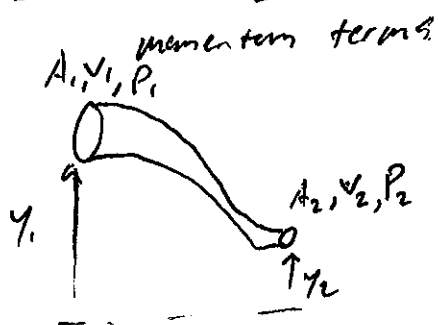


As fluid gets more confined, it flows faster!

Just like when you put your thumb over a water hose.

* Aorta Problem *

Bernoulli's Equation: Works like cons. of energy, but described in



$$\boxed{P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2}$$

Note: Imagine ~~different~~ $y_1 = y_2$

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 = \text{Const.}$$

What if we increased P_1 and v_1 can't change, but P_2 and v_2 can.

What happens if v_2 increases?

* Bernoulli Problem *

Airplane! Baseball! Cars Flipping!

Archimedes principle

Why was Archimedes so excited?

- He could measure volume of water displaced.

- Then mass.

- If the "density" of crown was less than gold density, the King had been fooled!

Prob 4:

Out: 

$$\cancel{\Sigma F_y = F_{air} - mg = 0}$$

$$\cancel{F_{air} = 1.2 \text{ N} = mg}$$

$$\Sigma F_y = 0$$

$$\Sigma F_y = F_T + F_B - mg = 0$$

$$F_B = mg - F_T = 1.3 \text{ N}$$

$$F_B = \rho_{oil} V_{oil} g = \rho_{oil} V_{ball} g$$

$$\rho_{oil} = \frac{F_B}{V_{ball} g}$$

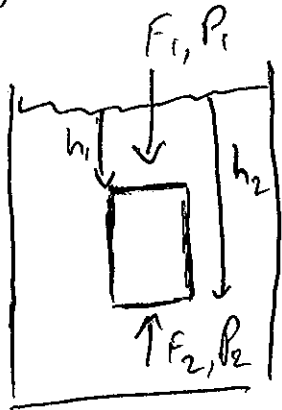
$$\rho_{ball} = \frac{m_{ball}}{V_{ball}} \Rightarrow V_{ball} = 1.6 \times 10^{-4} \text{ m}^3$$

$$\boxed{\rho_{oil} = 830 \text{ kg/m}^3}$$

Buoyancy

* Archimedes Story * - Only tell King/crown + bath part! "Find density!"

- Tendency of objects to feel "lighter" when submerged in a liquid like water.
- This occurs because the pressure pushing down on the top of the object is less than the pressure at the bottom of the object.



$$P_1 = P_A + \rho g h_1$$

$$P_2 = P_A + \rho g h_2$$

$$F_1 = P_1 A$$

$$F_2 = P_2 A$$

We say the buoyant force comes from the difference in these forces!

$$F_B = F_2 - F_1 = P_2 A - P_1 A = A(P_2 - P_1)$$

$$F_B = A(P_A + \rho g h_2 - P_A - \rho g h_1)$$

$$\boxed{F_B = A \rho g (h_2 - h_1)}$$

But notice! $A \cdot (h_2 - h_1) = A \Delta h = V$!

$$\boxed{F_B = \rho V g}$$

and $\rho = \frac{m}{V} \Rightarrow \rho V = m$

$$\boxed{F_B = m_F g}$$

← Shape doesn't matter!